Interactions of Heavy Charged Particles[1]

1. ENERGY-LOSS MECHANISMS

A heavy charged particle traversing matter loses energy primarily through the ionization and excitation of atoms. (Except at low velocities, a heavy charged particle loses a negligible amount of energy in nuclear collisions.) The moving charged particle exerts electromagnetic forces on atomic electrons and imparts energy to them. The energy transferred may be sufficient to knock an electron out of an atom and thus ionize it, or it may leave the atom in an excited, nonionized state. A heavy charged particle can transfer only a small fraction of its energy in a single electronic collision. Its deflection in the collision is negligible. Thus, a heavy charged particle travels an almost straight path through matter, losing energy almost continuously in small amounts through collisions with atomic electrons, leaving ionized and excited atoms in its wake.

2. MAXIMUM ENERGY TRANSFER IN A SINGLE COLLISION

In this section we calculate the maximum energy that a charged particle can lose in colliding with an atomic electron. We assume that the particle moves rapidly compared with the electron and that the energy transferred is large compared with the binding energy of the electron in the atom. Under these conditions the electron can be considered to be initially free and at rest, and the collision is elastic.

We assume a charged particle mass (mass M and velocity V_o) approaching an electron (mass m at rest). After the collision, which for maximum energy transfer is head-on, the particles move with speeds V_1 and v_1 along the initial line of travel of the incident particle. Since the total kinetic energy and momentum are conserved in the collision, we have the two relationships. [1]

$$\frac{1}{2}\mathrm{MV}^2 = \frac{1}{2}\mathrm{MV}_1^2 + \frac{1}{2}mv_1^2 \tag{1}$$

and

$$\mathrm{MV}_0 = \mathrm{MV}_1 + mv_1. \qquad \mathrm{V}_1 = \frac{MV_0 - Mv_1}{m} \tag{2}$$

If we solve Eq. (2) for v_1 and substitute the result into (1), we obtain

$$\mathrm{V}_1 = \frac{(M-m)V_o}{M+m} \tag{3}$$

Using this expression for V_1, we find for the maximum energy transfer

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[1] Much of this material is excerpted from James Turner's "Atoms, Radiation, and Radiation Protection", John Wiley and Sons, 1995. Chapter 5.

$$Q_{\max} = \frac{1}{2}MV_0^2 - \frac{1}{2}MV_1^2 = \frac{4mME}{(M = m)^2}, \tag{4}$$

Where E $= MV^2/2$ is the initial kinetic energy of the incident particle. All heavy charged particles travel essentially straight paths in matter.

Example
Calculate the maximum energy that a 10-MeV proton can lose in a single electronic collision.

Solution
Neglecting m compared with M, we have $Q_{\max} = 4mE/M = 4 \times 1 \times 10/1836 = 2.18 \times 10^{-2}$ MeV = 21.8 keV, which is only 0.22% of the proton's energy.

Table 1 below gives numerical results for a range of proton energies. Except at extreme relativistic energies, the maximum fractional energy loss for a heavy charged particle is small. It is a good approximation to calculate $Q_{\max}$ as though the struck electron were not bound, the collision then being elastic.

TABLE 1 Maximum Possible Energy Transfer, $Q_{\max}$, in Proton Collision with Electron

Proton Kinetic Energy E (MeV)	$Q_{\max}$ (MeV)	Maximum Percentage Energy Transfer $100Q_{\max}/E$
0.1	0.00022	0.22
1	0.0022	0.22
10	0.0219	0.22
100	0.229	0.23
10^3	3.33	0.33
10^4	136	1.4
10^5	1.06×10^4	10.6
10^6	5.38×10^5	53.8
10^7	9.21×10^6	92.1

3. STOPPING POWER

The average linear rate of energy loss of a heavy charged particle in a medium (expressed, for example, in MeV cm^{-1}) is of fundamental importance in radiation physics and dosimetry. This quantity, designated $-dE/dx$, is called the stopping power of the medium for the particle. It is also referred to as the linear energy transfer (LET) of the particle, usually expressed as keV μm^{-1} in water. Stopping power and LET are closely associated with the dose and with the biological effectiveness of different kinds of radiation.

The macroscopic cross section for a 1-MeV proton in water is 410 μm^{-1}.

The "macroscopic cross section",μ, is the probability per unit distance of travel that an electronic collision takes place. Its role in charged-particle penetration is analogous to that of the decay constant λ, which is the probability of disintegration per unit time in radioactive decay. We've seen that the reciprocal of the decay constant is equal to the mean life. In the same way, the reciprocal of μ is the mean distance of travel, or mean free path, of a charged particle between collisions. In the last example, the mean free path of the 1-MeV proton is $1/\mu = 1(410\ \mu m^{-1}) = 0.0024\ \mu m = 24$ Å. Atomic diameters are of the order of 1 Å to 2 Å.

Using relativistic quantum mechanics, Bethe derived the following expression for the stopping power of a uniform medium for a heavy charged particle:

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2 \beta^2} \left[In \frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2 \right]. \qquad (5)$$

In this relation

$k_0 = 8.99 \times 10^9$ N m^2 C^{-2} (recall Appendix C of earlier handout),
z = atomic number of the heavy particle,
e = magnitude of the electron charge,
n = number of electrons per unit volume in the medium,
m = electron rest mass,
c = speed of light in vacuum,
$\beta = V/c$ = speed of the particle relative to c,
I = mean excitation energy of the medium.

Only the charge ze and velocity V of the heavy charged particle enter the expression for stopping power. This fact is consistent with the universality of charged-particle energy-loss spectra in sudden collisions. For the medium, only the electron density n is important.

Figure 1 shows the stopping power of liquid water in MeV cm^{-1} for a number of charged particles as a function of their energy. The logarithmic term in Eq. 5 leads to an increase in stopping power at very high energies (as $\beta \rightarrow 1$), just discernable for muons in the figure. At low energies, the factor in front of the bracket in Eq. 5 increases as $\beta \rightarrow 0$.

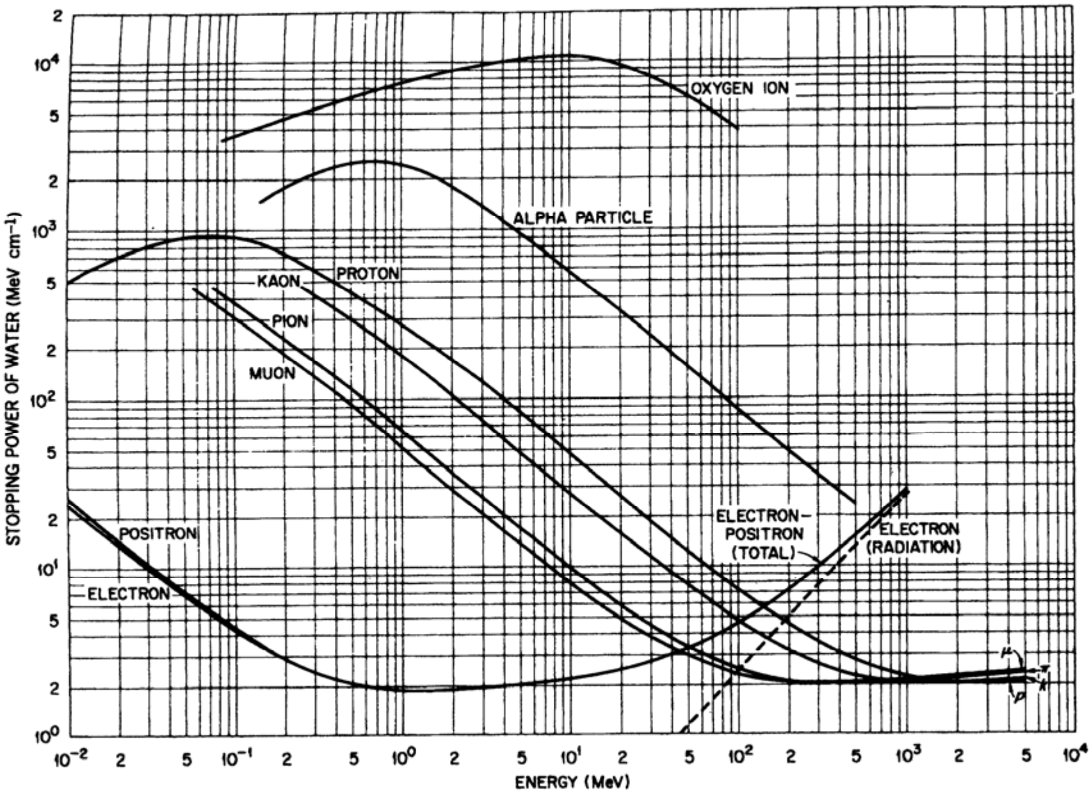


Figure 1 Stopping power of water in MeV cm^{-1} for va4rious heavy charged particles and beta particles. The muon, pion, and kaon are elementary particles with rest masses equal, respectively, to about 207, 270 and 967 electron rest masses.

However, the logarithm term then increases, causing a peak (called the Bragg peak) to occur. The linear rate of energy loss is a maximum there. This is exemplified in Figure 2 below which plots dE/dx of an alpha particle as a function of distance in a material. For most of the alpha particle track, the charge on the alpha is two electron charges, and the rate of energy loss increases roughly as 1/E as predicted by equation (5). Near the end of the track, the charge is reduced through electron pickup and the curve falls off. Plots are shown for both a single alpha particle track and for the average behavior of a parallel beam of alpha particles of the same initial energy. The two curves differ somewhat due to the effects of straggling. [Because the details of the microscopic interactions undergone by any specific particle vary somewhat randomly, its energy loss is a statistical or stochastic process. Therefore, a spread in energies always results after a beam of monoenergetic charged particles has passed through a given thickness of absorber. The width of this energy distribution is a measure of ‘energy straggling’, which varies with the distance along the particle track.]

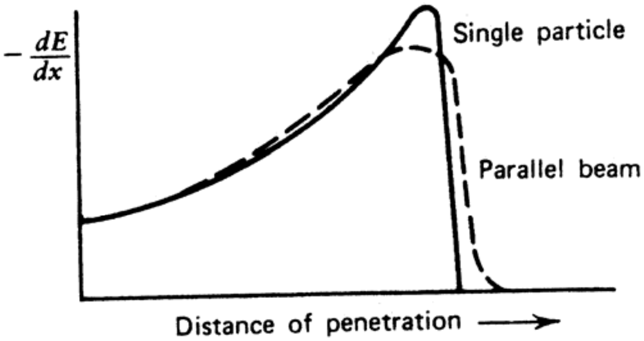


Figure 2. Rate of energy loss along an alpha particle track.

Mass Stopping Power: The mass stopping power of a material is obtained by dividing the stopping power by the density ρ. Common units for mass stopping power, -dE/ρdx, are MeV $\mathrm{cm}^2\ \mathrm{g}^{-1}$. The mass stopping power is a useful quantity because it expresses the rate of energy loss of the charged particle per g cm^{-2} of the medium traversed. In a gas, for example, -d*E*/dx depends on pressure, but –d*E*/ρdx does not, because dividing by the density exactly compensates for the pressure. In addition, the mass stopping power does not differ greatly for materials with similar atomic composition. For example, for 10-MeV protons the mass stopping power of H_2O is 45.9 MeV $\mathrm{cm}^2\ \mathrm{g}^{-1}$ and that of anthracene ($C_{14}H_{10}$) is 44.2 MeV $\mathrm{cm}^2\ \mathrm{g}^{-1}$. The curves in the figure above for water can be scaled by density and used for tissue, plastics, hydrocarbons, and other materials that consist primarily of light elements. For Pb (Z=82), on the other hand, -d*E*/ρdx = 17.5 MeV $\mathrm{cm}^2\mathrm{g}^{-1}$ for 10-MeV protons. Generally, heavy atoms are less efficient on a g cm^{-2} basis for slowing down heavy charged particles, because many of their electrons are too tightly bound in the inner shells to participate effectively in the absorption of energy.

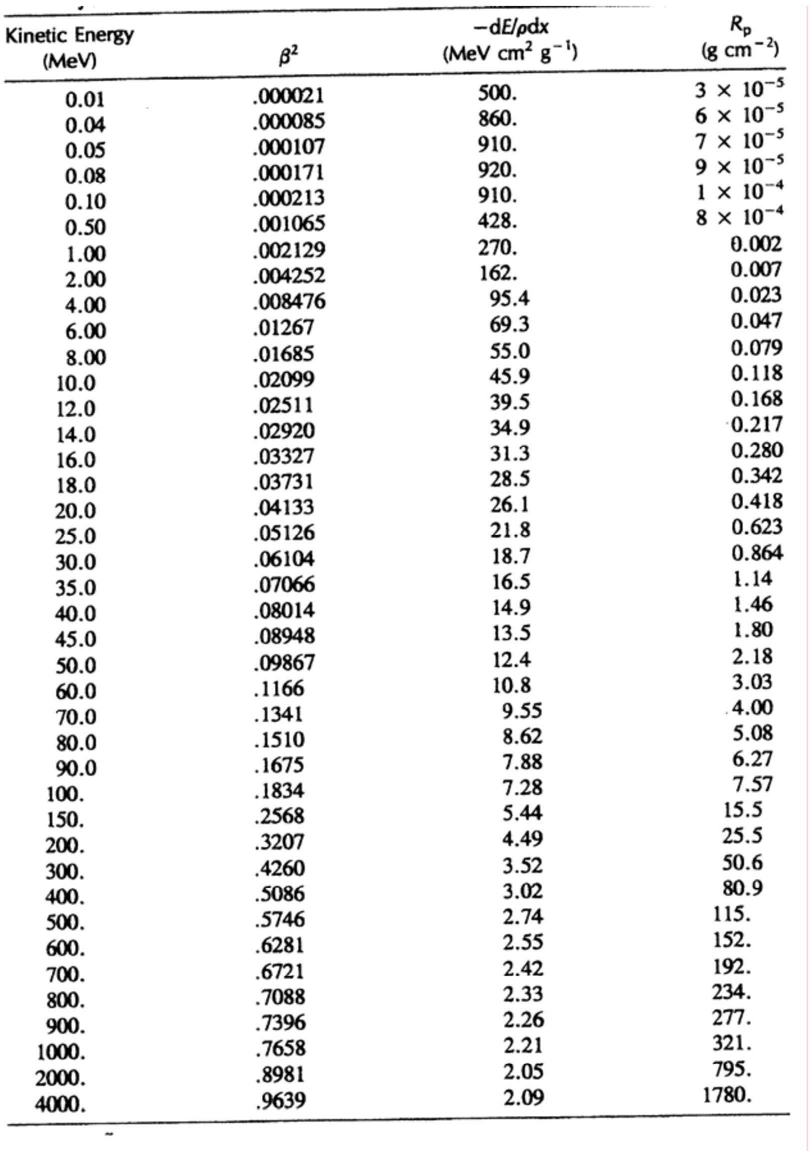
4. RANGE

The range of a charged particle is the distance it travels before coming to rest. The reciprocal of the stopping power gives the distance traveled per unit energy loss. Therefore, the range R(t) of a particle of kinetic energy T is the integral of this quantity down to zero energy:

$$\mathrm{R(T)} = \int_O^T \left(-\frac{dE}{dx}\right)^{-1} \mathrm{d}E. \tag{6}$$

Table 2 gives the mass stopping power and range of protons in water. The latter is expressed in g cm^{-2}; that is, the range in cm multiplied by the density of water (p = 1 g cm^{-3}). Like mass stopping power, the range in g cm^{-2} applies to all materials of similar atomic composition.

Table 2. Mass Stopping Power -*dE/ρdx* and Range R_p for Protons in Water



Kinetic Energy (MeV)	β^2	$-dE/\rho dx$ (MeV cm^2 g^{-1})	R_p (g cm^{-2})
0.01	.000021	500.	3×10^{-5}
0.04	.000085	860.	6×10^{-5}
0.05	.000107	910.	7×10^{-5}
0.08	.000171	920.	9×10^{-5}
0.10	.000213	910.	1×10^{-4}
0.50	.001065	428.	8×10^{-4}
1.00	.002129	270.	0.002
2.00	.004252	162.	0.007
4.00	.008476	95.4	0.023
6.00	.01267	69.3	0.047
8.00	.01685	55.0	0.079
10.0	.02099	45.9	0.118
12.0	.02511	39.5	0.168
14.0	.02920	34.9	0.217
16.0	.03327	31.3	0.280
18.0	.03731	28.5	0.342
20.0	.04133	26.1	0.418
25.0	.05126	21.8	0.623
30.0	.06104	18.7	0.864
35.0	.07066	16.5	1.14
40.0	.08014	14.9	1.46
45.0	.08948	13.5	1.80
50.0	.09867	12.4	2.18
60.0	.1166	10.8	3.03
70.0	.1341	9.55	4.00
80.0	.1510	8.62	5.08
90.0	.1675	7.88	6.27
100.	.1834	7.28	7.57
150.	.2568	5.44	15.5
200.	.3207	4.49	25.5
300.	.4260	3.52	50.6
400.	.5086	3.02	80.9
500.	.5746	2.74	115.
600.	.6281	2.55	152.
700.	.6721	2.42	192.
800.	.7088	2.33	234.
900.	.7396	2.26	277.
1000.	.7658	2.21	321.
2000.	.8981	2.05	795.
4000.	.9639	2.09	1780.

For two heavy charged particles at the same initial speed β, the ratio of their ranges is simply

$$\frac{R_1(\beta)}{R_2(\beta)} = \frac{z_2^2 M_1}{z_1^2 M_2}, \qquad (7)$$

where M_1 and M_2 are the rest masses and z_1 and z_2 are the charges. If particle number 2 is a proton ($M_2 = 1$ and $z_2 = 1$), then we can write for the range R of the other particle (mass $M_1 = M$ proton masses and charge $z_1 = z$)

$$R(\beta) = \frac{M}{z^2} R_p(\beta), \tag{8}$$

where $R_p(\beta)$ is the proton range.

Example
Use Table 2 to find the range of an 80-MeV $^3He^{2+}$ ion in soft tissue.

Solution
Applying Eq. 7 we have $z^2 = 4$, $M = 3$, and $R(\beta) = 3R_p(\beta)/4$. Thus the desired range is three-quarters that of a proton traveling with the speed of an 80-MeV $^3He^{2+}$ ion. At this speed, the proton has an energy of 80/3 = 26.7 MeV, that is, an energy smaller than that of the helium ion by the ratio of the masses. Interpolation in Table 2 gives for the proton range at this energy $R_p = 0.705$ g cm^{-2}. It follows that the range of the 80-MeV $^3He^2$ particle is (3/4) (0.0705) = 0.529 g cm^{-2}, or 0.529 cm in unit-density soft tissue.

Figure 3 shows the ranges in g cm^{-2} of protons, alpha particles, and electrons in water or muscle (virtually the same), bone, and lead. For a given proton energy, the range in g cm^{-2} is greater in Pb than in H_2O, consistent with the smaller mass stopping power of Pb.

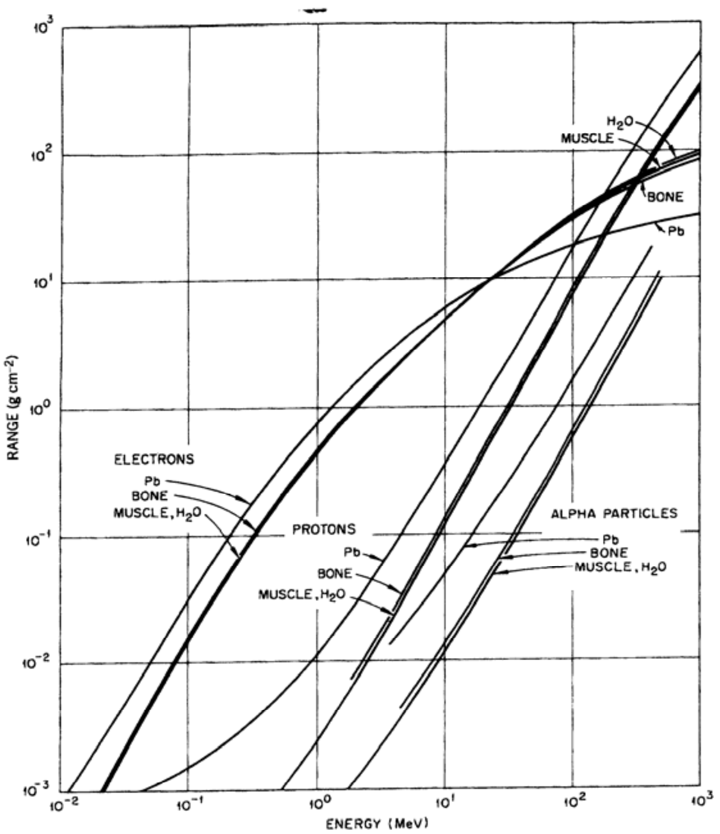


FIGURE 2 Ranges of protons, alpha particles, and electrons in water, muscle, bone, and lead, expressed in g cm^{-2}. (Courtesy Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the Department of Energy.)

Figure 4 gives the range in cm of protons, alpha particles, and electrons in air at standard temperature and pressure. For alpha particles in air at 15°C and 1-atm pressure, the following approximate empirical relations fir the observed range R in cm as a function of energy E in MeV:

$$R = 0.56E, \qquad E < 4; \tag{9}$$

$$R = 1.24E - 2.62, \qquad 4 < E < 8. \tag{10}$$

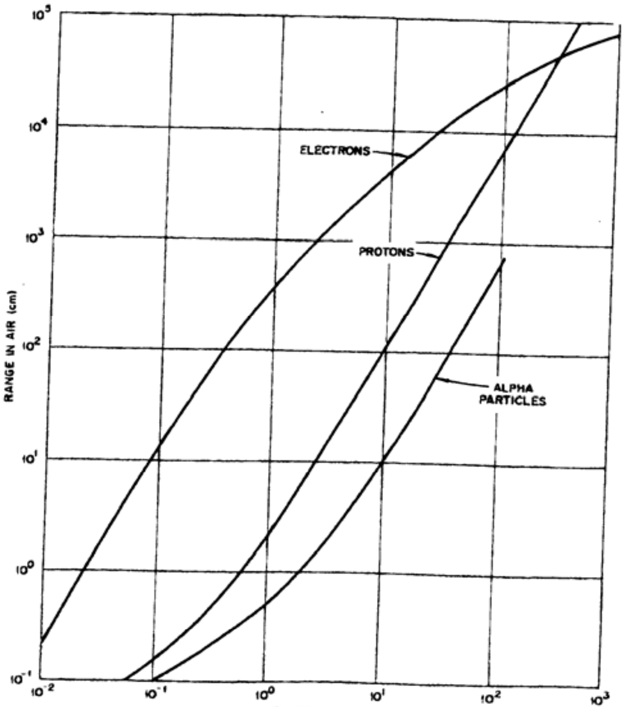


FIGURE 3 Ranges in cm of protons, alpha particles, and electrons in air at STP. (Courtesy Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the Department of Energy.)

Example

The radon daughter ${}^{214}_{84}$Po, which emits a 7.69-MeV alpha particle, is present in the atmosphere of uranium mines. What is the range of this particle in soft tissue?

Solution

We use the proton range in Table 2 to find the alpha-particle range in tissue. Applied to alpha rays, Eq. 8 gives ($z^2 = 4$ and $M = 4$) $R_\alpha(\beta) = R_p(\beta)$. Thus, the ranges of an alpha particle and a proton with the same velocity are the same. The ratio of the kinetic energies at the same speed is $T_\alpha/T_p = M_\alpha/M_p = 4$, and so $T_p = T_\alpha/4 = 7.69/4 = 1.92$ MeV. The alpha-particle range is, therefore, equal to the range of a 1.92-MeV proton. Interpolation in Table 5.3 gives $R_p = R_\alpha = 0.0066$ cm in tissue of unit density. The ^{214}Po alpha particles thus cannot penetrate the 0.007 cm minimum epidermal thickness from outside the body to reach living cells. On the other hand, inhaled particle matter containing ^{214}Po can be deposited in the lung. There the range of the alpha particles is sufficient to reach the basal cells of the bronchial epithelium. The increase in lung-cancer incidence among uranium miners over that normally expected has been linked to the alpha-particle dose from inhaled radon daughters.

Because of the statistical nature of energy losses by atomic collisions, all particles of a given type and initial energy do not travel exactly the same distance before coming to rest in a medium. This is refereed to as "range straggling". Thus, range as used here refers to the mean range, or average distance traveled.

5. SLOWING-DOWN TIME

We can use the stopping-power formula to calculate the rate at which a heavy charged particle slows down. the time rate of energy loss, -d*E*/d*t*, can be expressed in terms of the stopping power by using the chain of differentiation: -d*E*/d*t* = -(d*E*/dx)/(dt/dx) = v(-dE/dx), where V = dx/dt is the velocity of the particle. For a proton with kinetic energy T = 0.5 MeV in water, for example, the rate of energy loss is –d*E*/d*t* = 4.19 x 10^{11} MeV s^{-1}.

A rough estimate can be made of the time it takes a heavy charged particle to stop in matter, if one assumes that the slowing-down rate is constant. For a particle with kinetic energy *T*, this time is approximately,

$$\tau \sim \frac{T}{-dE/dt} = \frac{T}{V(-dE/dx)} . \tag{11}$$

For a 0.5-MeV proton in water, $\tau \sim$ (0.5 MeV)/(4.19 x 10^{11} MeV s^{-1}) = 1.2 x 10^{-12} s. Slowing-down rates and estimated stopping times for protons of other energies are given in Table 3. Because, as seen from Fig. 1, the stopping power increases as a proton slows down, actual stopping times are smaller than the estimates.

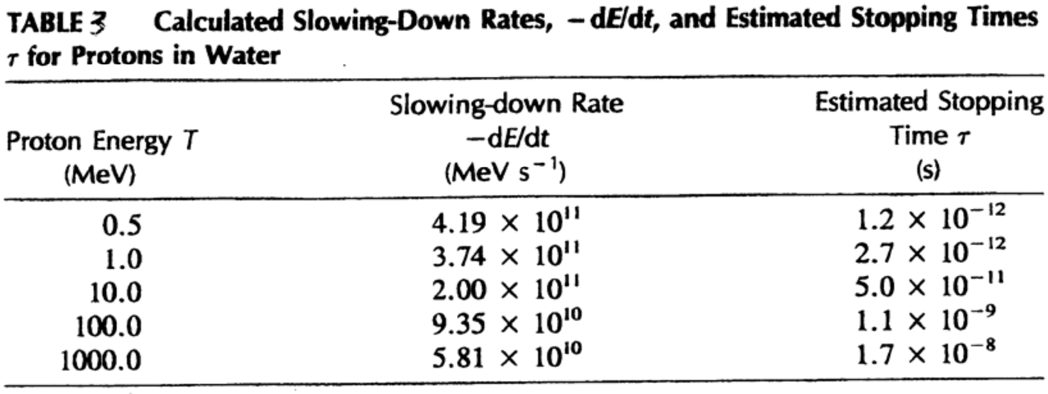


TABLE 3 Calculated Slowing-Down Rates, −d*E*/d*t*, and Estimated Stopping Times τ for Protons in Water

Proton Energy *T* (MeV)	Slowing-down Rate −d*E*/dt (MeV s^{-1})	Estimated Stopping Time τ (s)
0.5	4.19×10^{11}	1.2×10^{-12}
1.0	3.74×10^{11}	2.7×10^{-12}
10.0	2.00×10^{11}	5.0×10^{-11}
100.0	9.35×10^{10}	1.1×10^{-9}
1000.0	5.81×10^{10}	1.7×10^{-8}